Thus we see what interest there is for astronomy to determine the parallax of the sun by three independent methods ; I trust that the experiments that I have the honour to submit to the Academy will justify, by their precision, the theoretical importance of the physical method. R. M.

ON THE MUSCULAR MECHANICAL WORK DONE BEFORE EXHAUSTION*

II.

ONE of the principal sources of error in the series of experiments before discussed, was the fatigue caused by the downward-plunging weight. To eliminate this, the apparatus shown in vertical section in Figs. 1 and 2 was constructed. The shelf o, armed with a plate of car-spring caoutchouc, serves as a support for the weight. This shelf is fastened by the iron bands, I, and a vertical backpiece, B, to the slide, u, being further supported by a crosspiece. By suitable blocks, a, the slide may be raised to any desired height, which height is read off on a scale marked on the upright pieces, A. s is a support for the arm during the interval of rest, and it can be adjusted to any desired height ; W is a wire, from which a small cord, b, passes horizontally to the wall of the room. By some modifications this cord can also be made movable, which will become necessary when I come to investigate the influence of elevation of the arm, upon the work done. The weight is a bucket of shot, provided with a stiff bail and a wooden handle, so that for any position of the arm while lifting the weight, the line passing through the centre of the hand and the centre of gravity of the weight is a vertical. Placing the bucket upon the shelf o, the experimenter stands to
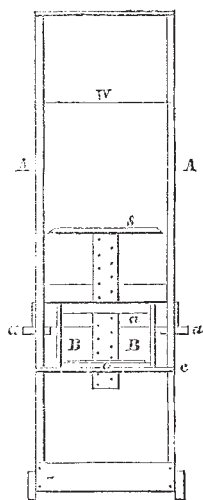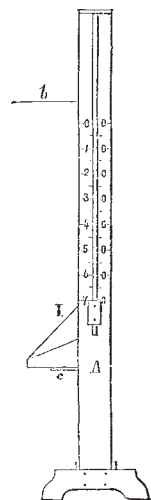

FIG. 1.   FIG. 2.

the right of the apparatus (as in Fig. 2), and lifts the weight until his knuckles touch the cord b. The instant of beginning and close of this interval of work is marked by the sharp click of a metronome, the time of whose beat is t. At the instant when the knuckles touch the string, the weight is grasped by an assistant, † and by him lowered to the shelf o, the arm of the experimenter being entirely relaxed, and resting upon the stiff bail of the bucket and the support s. ‡ This is continued until the arm becomes unable to lift the weight to the required height. The determination of the number of lifts should *never* be made by the experimenter, who should furthermore try to lose all estimate of time during the process. In the earlier experiments it became evident that the arm not only grew gradually stronger, but also that it varied greatly from day to day. In order to get some measure of the strength, the arm was exhausted on each day of experiment, by a constant weight (5·0 kilos.) lifted through a height $h = 0.70$ metres in a time $\tau = 1.25$ sec. The values of

n were all reduced to the mean strength, as shown by the constant experiment.

In the series here given, w was variable, $h = 0.70$ metres, and $\tau = 1.25$ sec., the interval of rest being equal to the interval of work. The mean value of n for the constant experiment for the weights $w = 3.0, 3.5, 4.0, \&c.—7.5$, in all 100 experiments, is 35·79. Taking these values, c, as the measure of the strength, and assuming that the work done with any weight at different times is proportional to the strength,* and we have—calling n' the number of lifts before exhaustion, and n the number reduced to the basis of the mean constant (35·79)—

$$n = \frac{35.79}{c} n'$$

from which we have the following values of n, which hereafter we shall call n (obs.) Each of these values is a mean of ten independent determinations.

TABLE II.

w	n (obs.)	n (calc.)	$d n$	e
2·50	283	242	− 14·4	7·5
3·00	152·5	150·3	− 1·4	3·7
3·50	95·8	99·4	+ 3·6	3·6
4·00	67·2	69·2	+ 2·9	2·9
4·50	51·2	50·1	− 2·1	3·3
5·00	36·9	37·4	+ 1·3	2·4
5·50	28·6	28·7	+ 0·3	2·0
6·00	22·7	22·5	− 0·9	1·3
6·50	18·1	18·0	− 0·5	1·1
7·00	14·5	14·6	+ 0·7	0·7
7·50	10·4	11·9	+14·4	0·9
8·00	7·7	9·9	+28·5	5·2

The determination of n for $w = 7.5$ and 8·0 was consciously bad, as the arm was unable to manage such weights at such a
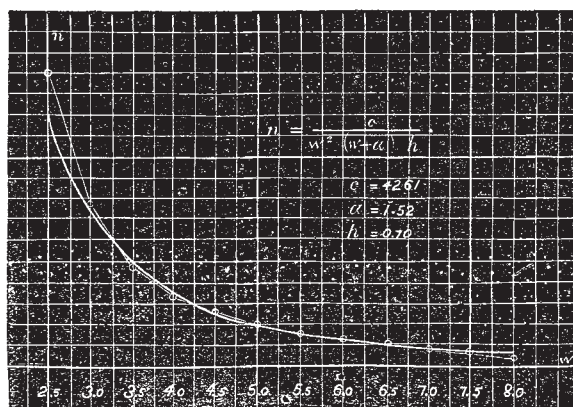

FIG. 3.

velocity, so that I was obliged to stop *before the arm was exhausted*.† The values of n for w less than 3·0 were also dropped in the final calculation, as with such light weights the work is found to vary greatly with a slight variation of strength.

Assuming the arm to be a uniform cylinder, and denoting by a one half the weight of the arm, and we have as the dynamical work done before exhaustion—

$$W = (w + \tfrac{1}{2}a) h n \qquad (6).$$

The value of a can be determined directly by means of a spring-balance. Exhaust the arm thoroughly, then grasp the hook of the spring-balance, the dial of which should be turned from the face of the experimenter, the reading being done by an assistant. After several minutes the muscles tire, and the practised experimenter can then gradually relax them fully. Untrained muscles, when thus tried, act involuntarily, and precise

results cannot be obtained. The mean of ten determinations gave, for my right arm, $a = 1·50$ kgr. The mean of twenty determinations likewise gave $a = 1·50$ kgr., with a probable error of 0·01 kgr. Calculating from (6) the values of W for the different values of w, and co-ordinating these two quantities, and it is plain that the function is hyperbolic. It was found that W did not vary inversely as $(w + a)$, or as any power of this quantity.* The equation

$$(w + a)\, h\, n = \frac{c}{w^v} \qquad (7).$$

was then assumed, where c and v are constants to be determined. From this we readily have

$$\log. (w + 1·5) + \log. n = k - v\log. w,$$

which is of the form

$$y = k - v.x,$$

where y and x can be calculated from the observations. Co-ordinating these values of y and x, and the curve is found to be linear, and we find v, as the change in y for each unit of change in x, to be 2·007. Hence Eq. (7) becomes

$$(w + a)\, h\, n = \frac{c}{w^2} \qquad (8).$$

Calculating now the values of a and c by the method of least squares, we find $c = 4261$ and $a = 1·52$. The difference between a (calc.) and a (obs.) is only 1·3 per cent. of a(obs.) Solving (8) for n, and substituting the proper values, and we have n (calc.), as given in Table II. $d\, n$ is the difference in per cent. of n (obs.) Column e is the probable error of n (obs.), also in per cent. The comparison between n (calc.) and n (obs.) is shown graphically in Fig. 3, the observations being represented by the small circles.

Soon after arriving at Eq. (8), Prof. Haughton's book came to hand, containing his reduction resulting in Eq. (5). As already shown, this equation does not represent my later and more accurate observations. In order to test the matter still further, experimentally, the following experiments were made :—

1. I lifted my right arm from a vertical to a horizontal ($h =$ 0·71 cm.), the experiments being conducted exactly as in the case of those given in Table II. The arm was lifted 2,000 times without feeling any appreciable exhaustion. According to (5), when $w = 0$, complete exhaustion should occur when $n = 1,000$. According to (8) it should occur when $n = \infty$.

2. A weight, $w = 0·5$ kgr., was lifted in the same manner, and the arm allowed to drop with the weight during the interval of rest, as in case of my earlier experiments. It was thus lifted 1,500 times with very little exhaustion. According to (5) complete exhaustion should occur when $n = 400$. According to (8) n should be 12,000. This would make the total time of exhaustion 8 hours and 20 minutes. The total mechanical work would be 16,800 kgr. metres. The daily labour of a working man is about 100,000 kgr. metres. From estimates based upon this fact, and from the slight fatigue felt in the second experiment, I am convinced that my arm, at its mean strength, could work for $8\frac{1}{3}$ hours at the above rate, if the experiment were conducted as described above, care being taken to eliminate the fatigue caused by standing on the feet, &c. It would, however, be a highly dangerous experiment.

It will be remembered that each value of n (obs.) in Table II. is a mean of ten independent determinations. It occurred to me to co-ordinate the originally observed values of n with the daily determination of strength c. The result was most instructive. Each value of w gave a curve which is really parabolic, but which—since one of these curves ($w = 5·0$) was taken as a unit in which to represent the others—appeared here as a straight line, or very nearly so, with exception of those which had been before rejected in calculating the constants. The reason for the great value of n for $w = 2·5$ (Table II.) is thus apparent.

This at once opened up a new field—the relation of strength to work. In the investigations here the strength is determined by a spring balance, so arranged that the arm is held horizontally and the strain exerted upwards. Calling s the reading of the dynanometer, and the strength is $(w + a)$.† Co-ordinating, for the different weights used in Table II., the strength with the work done before exhaustion, and we have for each value of w a curve which is apparently parabolic, intersect-

* The equation $(w + a)\, h\, n = \dfrac{c}{(w + a)\, 2·63}$ will represent the observations, but it is a highly improbable relation

† $(s + a)$ is really the highest tension attainable by the muscle in exerting a uniformly accelerated force, with a uniform velocity through the space moved over by the hook of the dynanometer.

ing the axis of abscissæ (strength) at a point just inside the point where $s + a = w + a$.* As w diminishes, the curves increase in steepness with great rapidity. Eq. (8) shows the relation between the points on each of these curves, which correspond to my mean strength.

This opens up a way of estimating the statical work ot a muscle, a problem which has been in view from the outset. We will take as the unit of statical work, the *kilogram-second*, or the work done by a muscle in sustaining for one second a strain of one kilo. exerted at right angles to its line of contraction. If now the same weights be used in exhausting the horizontally outstretched arm, we shall have by co-ordinating the work (in kgr.-sec.) with the strength, a system of curves as in the case of dynamical work. Accurate values of the constants for these curves have not yet been obtained, and we therefore will not discuss them further here. For each weight, co-ordinate the dynamical with the statical work, and it is readily seen that the relation between them can be made out, so that—given the total energy of a muscle in kgr.-sec. with any weight, and we can calculate the dynamical work in kgr-metres which this same muscle could do with this same weight. I intend to determine as accurately as possible the values of the constants in the cases heretofore discussed in these papers. I shall also thus investigate the effect of variation of the angle of elevation of the arm on the dynamical and statical work, including the case of statical work where the angle of elevation is zero : also the dynamical work, where the strain on the muscles is *continuous*, and (1) where the strain on the muscles (a weight) is constant, and the velocity of motion uniformly varied ; (2) where the velocity is constant, and the weight uniformly varied ; and (3) where both weight and velocity are constant. Making in this latter case, $v = 0$, and we have the case of statical work. The apparatus necessary for this investigation has been already devised. FRANK E. NIPHER

SCIENTIFIC SERIALS

Zeitschrift der Oesterreichischen Gesellschaft für Meteorologie, Dec. 15.—To this number Dr. Prestel contributes an article on lines of cirrus as a means of foretelling storms. Storm signals he presumes to be inadequate for warning sailors of an approaching gale. He has compared during last year the indications of cirrous streaks with the weather shown by the charts to be prevalent on each day when his observations were made. From all the instances in which the streaks were well developed, he comes to the conclusion that the currents of the upper air do not follow the law of Buys Ballot ; that is, that in the region of cirrus the air has neither a cyclonic nor anticyclonic movement, but streams from the point of highest pressure in the area of high pressure to the point of lowest pressure in the area of low pressure.—Herr Köppen, having remarked the tendency of cyclones to follow closely upon one another, gives a table for Northern Russia of the intervals which most commonly separate them. Of 107 cyclones, occupying 393 days in the territory, 33 per cent. came in less than twenty-four hours after their predecessors ; 32 per cent. after an interval of one day ; 19 per cent. after two or three days ; 19 per cent. after four, five, or six days ; and 18 per cent. after seven, eight, nine, or ten days.—The observations of MM. Fautrat and Sartiaux, by which it appeared that more rain fell within than without the forest of Halatte, are objected to on account of the disturbing influence of wind, which blows less stongly at the one position, six metres above tree-tops, than at the other, fifteen metres above the plain.

Reale Istituto Lombardo. Rendiconti : vol. vii. fasc. ix., xi.— The first paper is On variations in the temperature of Milan, by Giovanni Celoria. Meteorological observations were commenced at the Observatory of Brera in 1763, and have been carried on without intermission, and show regular and irregular variations. The maximum temperature follows the culmination of the sun, and shows an oscillation in time of seventy minutes, being at 2h. 4m. in January and at 3h. 14m. in July. The minimum temperature in summer is eight minutes before the rising of the sun, and in winter forty-nine minutes before sunrise. This variation is less at Milan than elsewhere. The author follows Dove in dividing the year into seventy-three periods of five days each. There are two periods of medium temperature in the year, April 15 and 16, and October 18 ; 179 days are colder and 186 hotter

* My left arm is about five-sixths the [strength of the right. Each varies greatly from day to day. Several other persons, the length of whose bones approximated my own, have been experimented upon. The co-oridinated values of work and strength are continuous with my own.